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PRE-BOARD EXAMINATION - 2015-16

Subject: Mathematics (Class XII)
(Solutions).

Section - A

Q1 to 6 1 mk for correct Ans

1) $|adj(adj A)| = |adj A|^{n-1}$
$= |A|^{(n-1)(n-1)}$
$= |A|^{(n-1)^2}$
$= 4^{(3-1)^2}$
$= 4^4 = 256$

2) $y = c_1 e^{c_2 x}$ — (1)
$\Rightarrow \frac{dy}{dx} = c_1 c_2 e^{c_2 x}$
$\Rightarrow \frac{dy}{dx} = c_2 y$ — (2)
Again differentiating b/s w.r.t x:
$\frac{d^2y}{dx^2} = c_2 \frac{dy}{dx}$ — (3)
From eq (2) & (3):
$\frac{d^2y}{dx^2} = \frac{1}{y}\left(\frac{dy}{dx}\right)^2$
which is the reqd D.E.

3) Order $\to 2$ & degree $\to 2$
$\therefore$ Sum of order & degree $= 2+2 = 4$ Ans

4) Here $\vec{a}, \vec{b}, \vec{c}$ are coplanar
$\Rightarrow \vec{a}\cdot(\vec{b}\times\vec{c}) = 0$
$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ x & x-2 & -1 \end{vmatrix} = 0$
$\Rightarrow 1(1-2x+4) - 1(-1-2x) + 1(x-2+x) = 0$
$\Rightarrow (5-\cancel{2x}) + 1 + \cancel{2x} + 2x - 2 = 0$
$\Rightarrow 4 + 2x = 0$
$\Rightarrow x = -4/2 = -2$ Ans.

5) $\vec{b}$ is a zero vector.

6) Let the reqd plane be $x - 2y + 2z + k = 0$ — (1)
$\perp$ dist of (1) from $(0,0,0) = 1$ unit
$\Rightarrow \frac{|k|}{\sqrt{(1)^2+(-2)^2+(2)^2}} = 1$
$\Rightarrow \frac{|k|}{3} = 1$
$\Rightarrow \frac{\pm k}{3} = 1$
$\Rightarrow k = \pm 3$
$\therefore$ Reqd plane is $x - 2y + 2z + 3 = 0$ or $x - 2y + 2z - 3 = 0$.

Teacher's Signature :

Section - B

7) $I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -\tan\alpha/2 \\ \tan\alpha/2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & \tan\alpha/2 \\ -\tan\alpha/2 & 1 \end{bmatrix}$

$(I - A)^{-1} = \dfrac{1}{(1+\tan^2\alpha/2)} \begin{bmatrix} 1 & -\tan\alpha/2 \\ +\tan\alpha/2 & 1 \end{bmatrix} = \dfrac{1}{\sec^2\alpha/2} \begin{bmatrix} 1 & -\tan\alpha/2 \\ +\tan\alpha/2 & 1 \end{bmatrix}$

$(I + A) = \begin{bmatrix} 1 & -\tan\alpha/2 \\ \tan\alpha/2 & 1 \end{bmatrix}$ — (2)

From (1) & (2) : $(I+A)(I-A)^{-1} = \dfrac{1}{\sec^2\alpha/2} \begin{bmatrix} 1 & -\tan\alpha/2 \\ \tan\alpha/2 & 1 \end{bmatrix} \begin{bmatrix} 1 & -\tan\alpha/2 \\ \tan\alpha/2 & 1 \end{bmatrix}$

$= \dfrac{1}{\sec^2\alpha/2} \begin{bmatrix} 1-\tan^2\alpha/2 & -2\tan\alpha/2 \\ 2\tan\alpha/2 & 1-\tan^2\alpha/2 \end{bmatrix}$

$= \begin{bmatrix} \cos^2\alpha/2 - \sin^2\alpha/2 & -2\sin\alpha/2\cos\alpha/2 \\ 2\sin\alpha/2 & \cos^2\alpha/2 - \sin^2\alpha/2 \end{bmatrix}$

$= \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ Ans.

8) Buildings

$$\begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{cc} \text{Stone Trucks} & \text{Sand Trucks} \end{array} \\ \begin{bmatrix} 35 & 14 \\ 30 & 8 \\ 29 & 9 \end{bmatrix}_{3\times2} \end{array} \begin{array}{c} \text{Cost per truck} \\ \begin{bmatrix} 1000 \\ 400 \end{bmatrix}_{2\times1} \end{array} \begin{array}{l} \to \text{Stones Truck} \\ \to \text{Sand Truck} \end{array} = \begin{array}{c} \text{Amt. rcd from each firm} \\ \begin{bmatrix} 40,600 \\ 33,200 \\ 32,600 \end{bmatrix}_{3\times1} \end{array} \begin{array}{l} \text{firm A} \\ \text{firm B} \\ \text{firm C} \end{array}$$

$\therefore$ Total amount spent by the contractor $= 40,600 + 33,200 + 32,600$
$= ₹1,06,400/-$

Building contractor should possess the following values:
- Honesty
- Punctuality OR any other appropriate value

9) $L.H.S = \begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & a+b & c \end{vmatrix}$

$C_1 \to aC_1,\ C_2 \to bC_2,\ C_3 \to cC_3$

$L.H.S = \dfrac{1}{abc} \begin{vmatrix} a^2 & b^2-bc & c^2+bc \\ a^2+ac & b^2 & c^2-ac \\ a^2-ab & ab+b^2 & c^2 \end{vmatrix}$

$C_1 \to C_1 + C_2 + C_3$

$L.H.S = \dfrac{1}{abc} \begin{vmatrix} a^2+b^2+c^2 & b^2-bc & c^2+bc \\ a^2+b^2+c^2 & b^2 & c^2-ac \\ a^2+b^2+c^2 & ab+b^2 & c^2 \end{vmatrix}$

~~Taking $(a^2+b^2+c^2)$ common from C_1 we get:~~

$$L.H.S = \frac{1}{abc}(a^2+b^2+c^2)\begin{vmatrix} 1 & b^2-bc & c^2+bc \\ 1 & b^2 & c^2-ac \\ 1 & ab+b^2 & c^2 \end{vmatrix}$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$\therefore L.H.S = \frac{(a^2+b^2+c^2)}{abc}\begin{vmatrix} 1 & b^2-bc & c^2+bc \\ 0 & bc & -ac-bc \\ 0 & ab+bc & -bc \end{vmatrix}$$

Expanding along C_1, we get:

$$L.H.S = \frac{(a^2+b^2+c^2)}{abc}\begin{vmatrix} bc & -ac-bc \\ ab+bc & -bc \end{vmatrix}$$

Taking b common from C_1 & c common from C_2 we get:

$$L.H.S = \frac{(a^2+b^2+c^2)}{abc} \times bc \begin{vmatrix} c & -a-b \\ a+c & -b \end{vmatrix}$$

$$\Rightarrow L.H.S = \frac{1}{a}(a^2+b^2+c^2)\{-bc + a^2 + ab + ac + bc\}$$

$$= \frac{1}{a}(a^2+b^2+c^2) \times a(a+b+c)$$

$$\Rightarrow L.H.S = (a+b+c)(a^2+b^2+c^2) = R.H.S \quad \text{Hence Proved}$$

10) $(\tan^{-1}x)^2 + \left(\frac{\pi}{2} - \tan^{-1}x\right)^2 = \frac{5\pi^2}{8}$

$$\Rightarrow (\tan^{-1}x)^2 + \frac{\pi^2}{4} - \pi\tan^{-1}x + (\tan^{-1}x)^2 = \frac{5\pi^2}{8}$$

$$\Rightarrow 2(\tan^{-1}x)^2 - \pi\tan^{-1}x = \frac{5\pi^2}{8} - \frac{\pi^2}{4} = \frac{5\pi^2 - 2\pi^2}{8} = \frac{3\pi^2}{8}$$

Let $\tan^{-1}x = t$

$$\Rightarrow 2t^2 - \pi t - \frac{3\pi^2}{8} = 0$$

$$\Rightarrow t = \frac{\pi \pm \sqrt{\pi^2 - 4(2)\left(\frac{-3\pi^2}{8}\right)}}{4}$$

Teacher's Signature :

$$\Rightarrow t = \frac{\pi \pm \sqrt{4\pi^2}}{4}$$

$$\Rightarrow t = \frac{\pi \pm 2\pi}{4}$$

$$\Rightarrow t = \frac{3\pi}{4}, \frac{-\pi}{4}$$

$\Rightarrow \tan^{-1}x = \frac{3\pi}{4}$ or $\tan^{-1}x = -\frac{\pi}{4}$

$\Rightarrow x = \tan\left(\frac{3\pi}{4}\right)$ or $x = \tan(-\pi/4)$

$\Rightarrow x = \tan(\pi - \pi/4)$ or $x = -\tan\pi/4$

$\Rightarrow x = -1$ or $x = -1$

$\therefore$ Ans: $\boxed{x = -1}$

11) At $x = 2$

$L.H.L = \lim_{x \to 2^-} (2 - x) = 2 - 2 = 0$

$R.H.L = \lim_{x \to 2^+} (-2 + 3x - x^2) = -2 + 6 - 4 = 0.$

$L.H.L = R.H.L$ at $x = 2 \Rightarrow f$ is continuous at $x = 2$

Further, $L.H.D = \lim_{h \to 0^-} \frac{f(2-h) - f(2)}{-h}$

$= \lim_{h \to 0^-} \frac{(2 - (2-h)) - (0)}{-h} = \lim_{h \to 0^-} \frac{h}{-h} = \lim_{h \to 0^-} (-1) = \underline{-1}$

$R.H.D = \lim_{h \to 0^-} \frac{f(2+h) - f(2)}{h}$

$= \lim_{h \to 0^-} \frac{(-2 + 3(2+h) - (2+h)^2) - (0)}{h}$

$= \lim_{h \to 0^-} \frac{(-2 + 6 + 3h - 4 - h^2 - 4h)}{h}$

$= \lim_{h \to 0^-} \frac{(-h^2 - h)}{h} = \lim_{h \to 0^-} (-h - 1) = \underline{-1}$

$L.H.D = R.H.D$ at $x = 2 \Rightarrow f$ is differentiable at $x = 2$.

$\therefore f$ is continuous & differential at $x = 2$. Ans.

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12) $y = x^x$ — (1)

Taking log of b/s, $\log_e y = x \log_e x$ — (2)

Differentiating b/s w.r.t x we get:

$$\frac{1}{y}\frac{dy}{dx} = x \cdot \frac{1}{x} + 1 \cdot \log_e x$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = 1 + \log_e x \quad - (3)$$

Again differentiating b/s w.r.t. x:

$$\frac{-1}{y^2}\left(\frac{dy}{dx}\right)^2 + \frac{1}{y}\frac{d^2y}{dx^2} = \frac{1}{x}$$

$$\Rightarrow \cancel{-\left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2} = \frac{y}{x}}$$

$$\Rightarrow \cancel{xy\frac{d^2y}{dx^2} - x\left(\frac{dy}{dx}\right)^2 - y = 0}$$

$$\Rightarrow \frac{-1}{y}\left(\frac{dy}{dx}\right)^2 + \left(\frac{d^2y}{dx^2}\right) = \frac{y}{x}$$

$$\Rightarrow \frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$$ Hence Proved.

13) $$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1 + 1/t^2}{1 - 1/t^2} = \frac{t^2+1}{t^2-1}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{t^2+1}{t^2-1}\right) \times \frac{dt}{dx}$$

$$= \frac{(t^2-1)(2t) - (t^2+1)(2t)}{(t^2-1)^2} \times \frac{1}{1 - 1/t^2}$$

$$= \frac{\cancel{2t^3} - 2t - \cancel{2t^3} - 2t}{(t^2-1)^2} \times \frac{t^2}{t^2-1}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-4t^3}{(t^2-1)^3}$$ Hence Proved.

Teacher's Signature : _____

14) $\int_{-1}^{3} \{|x-2| + [x]\}\, dx$.

$= \int_{-1}^{0} (-x+2-1)\,dx + \int_{0}^{1} (-x+2+0)\,dx + \int_{1}^{2} (-x+2+1)\,dx + \int_{2}^{3} (x-2+2)\,dx$

$= \int_{-1}^{0} (1-x)\,dx + \int_{0}^{1} (2-x)\,dx + \int_{1}^{2} (3-x)\,dx + \int_{2}^{3} x\,dx$

$= \left(x - \frac{x^2}{2}\right)_{-1}^{0} + \left(2x - \frac{x^2}{2}\right)_{0}^{1} + \left(3x - \frac{x^2}{2}\right)_{1}^{2} + \left(\frac{x^2}{2}\right)_{2}^{3}$.

$= (0+1) - \frac{1}{2}(0-1) + 2(1-0) - \frac{1}{2}(1-0) + 3(2-1) - \frac{1}{2}(2^2-1^2) + \frac{1}{2}(3^2-2^2)$

$= 1 + \frac{1}{2} + 2 - \frac{1}{2} + 3 - \frac{1}{2}(3) + \frac{1}{2}(5)$

$= 6 - \frac{3}{2} + \frac{5}{2} = 6+1 = 7$. Ans.

15) $I = \int_{0}^{\pi/2} \frac{\sin 2x}{\sin^4 x + \cos^4 x}\, dx$.

↓

Let $\sin^2 x = t \Rightarrow 2\sin x \cos x\, dx = dt$

$\Rightarrow \sin 2x\, dx = dt$

when $x = 0$, $t = 0$

$x = \pi/2$, $t = 1$

$\therefore I = \int_{0}^{1} \frac{dt}{t^2 + (1-t)^2}$

$\Rightarrow I = \int_{0}^{1} \frac{dt}{2t^2 - 2t + 1} = \frac{1}{2} \int_{0}^{1} \frac{dt}{t^2 - t + 1/2}$

$+1/4 - 1/4$

$\therefore I = \frac{1}{2} \int_{0}^{1} \frac{dt}{\left(t - \frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$

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$\Rightarrow I = \frac{1}{2} \times \frac{1}{\frac{1}{2}} \tan^{-1}\left(\frac{t - 1/2}{1/2}\right)_0$

$\Rightarrow I = \tan^{-1}(2t-1)_0^1$

$\Rightarrow I = [\tan^{-1}(1) - \tan^{-1}(-1)]$

$\Rightarrow I = \frac{\pi}{4} + \tan^{-1}(1)$

$\Rightarrow I = \pi/4 + \pi/4 = \boxed{\pi/2}$ Ans.

16) $I = \int \frac{x^2}{(x\sin x + \cos x)^2} dx$

$= \int \frac{x}{\cos x} \cdot \frac{x\cos x}{(x\sin x + \cos x)^2} dx$

(I) (II)

Rough:
Let $x\sin x + \cos x = t$
$\Rightarrow (x\cos x + \sin x - \sin x)dx = dt$

$\therefore I = x\sec x \int \frac{x\cos x}{(x\sin x + \cos x)^2} dx - \int \left(\frac{d}{dx}(x\sec x) \int \frac{x\cos x\, dx}{(x\sin x + \cos x)^2}\right) dx$

Let $x\sin x + \cos x = t$
$\Rightarrow (x\cos x + \sin x - \sin x)dx = dt$

$\therefore I = x\sec x \int \frac{dt}{t^2} - \int (\sec x + x\sec x\tan x) \int \frac{dt}{t^2} dx$

$\Rightarrow I = x\sec x\left(\frac{-1}{t}\right) - \int \left(\frac{1}{\cos x} + \frac{x\sin x}{\cos x)^2}\right)\left(\frac{-1}{t}\right) dx$

$\Rightarrow I = \frac{-x\sec x}{(x\sin x + \cos x)} + \int \frac{(\cos x + x\sin x)}{\cos^2 x} \times \frac{1}{(x\sin x + \cos x)} dx$

$\Rightarrow I = \frac{-x\sec x}{(x\sin x + \cos x)} + \int \sec^2 x\, dx$

$\Rightarrow I = \frac{-x\sec x}{(x\sin x + \cos x)} + \tan x + C$ Ans.

Teacher's Signature :

OR

$$I = \int \frac{\cos 2x \sin 4x}{\cos^4 x (1+\cos 2x)}\, dx$$

$$\Rightarrow I = \int \frac{\cos 2x \sin 4x \, dx}{(\cos^2 x)^2 (1+\cos 2x)} = \int \frac{\cos 2x \sin 4x \, dx}{\left(\frac{1+\cos 2x}{2}\right)^2 (1+\cos 2x)}$$

$$\Rightarrow I = \int \frac{4 \cos 2x \sin 4x \, dx}{(1+\cos 2x)^3} = \int \frac{8 \cos 2x \cos 2x \sin 2x \, dx}{(1+\cos 2x)^3}$$

Let $\cos 2x = t$

$\Rightarrow -2 \sin 2x \, dx = dt$

$\therefore I = -4 \int \frac{t^2 \, dt}{(1+t)^3}$

$\Rightarrow I = -\int \frac{4t^2}{(1+t)^3}\, dt$

Let $\frac{4t^2}{(1+t)^3} = \frac{A}{1+t} + \frac{B}{(1+t)^2} + \frac{C}{(1+t)^3}$

$\Rightarrow (4t^2) = A(1+t)^2 + B(1+t) + C$

Putting $t = -1$, $4 = C$

$t = 0$, $0 = A + B + 4 \Rightarrow A + B = -4$ — (1)

$t = 1$, $4 = 4A + 2B + 4 \Rightarrow 4A + 2B = 0$

or $2A + B = 0$

or $B = -2A$ — (2)

Using (2) in (1): $-A = -4$

or $A = 4$ — (3)

$\therefore B = -8$ — (4)

$$\therefore I = -\left[4 \int \frac{dt}{1+t} - 8 \int \frac{dt}{(1+t)^2} + 4 \int \frac{dt}{(1+t)^3}\right]$$

$$\Rightarrow I = -4 \ln|1+t| + 8\left(\frac{-1}{1+t}\right) + 4\left(\frac{(1+t)^{-2}}{-2}\right) + C$$

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or $I = -4\ln|1+\cos 2x| - \frac{8}{1+\cos 2x} - \frac{2}{(1+\cos 2x)^2} + C$ Ans.

17) Given $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$\Rightarrow \vec{a}+\vec{b}=-\vec{c}$

$\Rightarrow (\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b})=(-\vec{c})\cdot(-\vec{c})$

$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+2\vec{a}\cdot\vec{b}=|\vec{c}|^2$

$\Rightarrow 9+25+2|\vec{a}||\vec{b}|\cos\theta = 49$ where θ is the $\angle$ between $\vec{a}$ & $\vec{b}$.

$\therefore 2\times 3\times 5\cos\theta = 49-9-25$

$\Rightarrow \cos\theta = \frac{15}{30} = \frac{1}{2}$

$\Rightarrow \theta = \cos^{-1}(1/2) = 60^\circ$

$\therefore$ $\angle$ bet $\vec{a}$ & $\vec{b}$ is $\pi/3$ Ans.

OR

$|(\vec{a}\times\vec{b})\cdot\vec{c}| = |\vec{a}||\vec{b}||\vec{c}|$

$\Leftrightarrow |\,|\vec{a}\times\vec{b}||\vec{c}|\cos\theta\,| = |\vec{a}||\vec{b}||\vec{c}|$

$\Leftrightarrow |\,|\vec{a}||\vec{b}|\sin\alpha\,|\vec{c}|\cos\theta\,| = |\vec{a}||\vec{b}||\vec{c}|$

$\Leftrightarrow |\sin\alpha\cos\theta| = 1$

$\Leftrightarrow |\sin\alpha||\cos\theta| = 1$

$\Leftrightarrow \alpha = \pi/2$ & $\theta = 0$

$\Leftrightarrow \vec{a}\perp\vec{b}$ & $\vec{a}\times\vec{b}\parallel\vec{c}$

i.e $\vec{c}\perp$ to both $\vec{a}$ & $\vec{b}$

$\Leftrightarrow \vec{a},\vec{b},\vec{c}$ are mutually $\perp$

$\Leftrightarrow \vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0$ Hence Proved.

18)

Let l, m, n be the d.c's of the line

$\therefore l = \cos\alpha = m = n$

$\Rightarrow l^2 + m^2 + n^2 = 1$

$\Rightarrow 3\cos^2\alpha = 1$

$\Rightarrow \cos^2\alpha = 1/3 \Rightarrow \cos\alpha = 1/\sqrt{3} = l = m = n$

$\therefore$ Line is: $\frac{x-2}{1/\sqrt{3}} = \frac{y+1}{1/\sqrt{3}} = \frac{z-2}{1/\sqrt{3}}$ — (1)

$\Rightarrow \frac{x-2}{1} = \frac{y+1}{1} = \frac{z-2}{1} = \lambda$ (say)

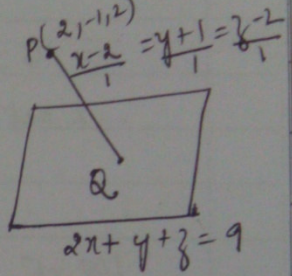
Any gen. pt on the above line is: $(\lambda+2, \lambda-1, \lambda+2)$ → say Q.

$\because$ Q lies on the plane $2x+y+z=9$

$\Rightarrow 2(\lambda+2) + \lambda - 1 + \lambda + 2 = 9$

$\Rightarrow 4\lambda + 5 = 9 \Rightarrow 4\lambda = 4 \Rightarrow \lambda = 1$

$\therefore$ Pt. Q is: $(3, 0, 3)$

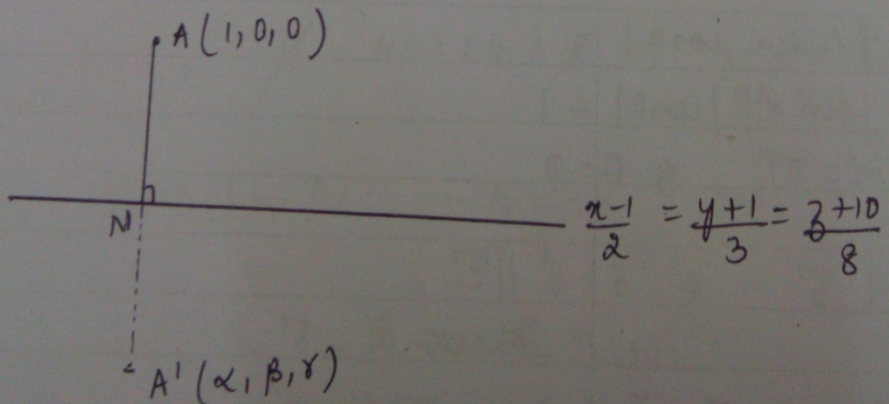


$\therefore |PQ| = \sqrt{(3-2)^2 + (0+1)^2 + (3-2)^2}$

$= \sqrt{1+1+1}$

$= \sqrt{3}$ units Ans.

OR



Given line is: $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z+10}{8} = \lambda$ (say) — (1)

$\therefore$ Any gen pt (say N) on (1) is: $(2\lambda+1, 3\lambda-1, 8\lambda-10)$

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$\therefore$ dr's of AN: $2\lambda, 3\lambda-1, 8\lambda-10$

AN $\perp$ line (1)

$\Rightarrow 2(2\lambda)+3(3\lambda-1)+8(8\lambda-10)=0$

$\Rightarrow 4\lambda+9\lambda+64\lambda-3-80=0$

$\Rightarrow 77\lambda = 83$

$\Rightarrow \lambda = \frac{83}{77}$

$\therefore$ Pt N is: $\left(2\times\frac{83}{77}+1,\ 3\left(\frac{83}{77}\right)-1,\ 8\left(\frac{83}{77}\right)-10\right)$

i.e $\left(\frac{243}{77}, \frac{172}{77}, \frac{-106}{77}\right)$

For pt. A':

N is mid-pt of AA'

$\Rightarrow \frac{243}{77} = \frac{1+\alpha}{2} \Rightarrow \alpha = 2\left(\frac{243}{77}\right)-1 = \frac{409}{77}$

& $\frac{172}{77} = \frac{0+\beta}{2} \Rightarrow \beta = \frac{344}{77}$

& $\frac{-106}{77} = \frac{0+\gamma}{2} \Rightarrow \gamma = -\frac{212}{77}$

$\therefore$ Reqd reflection is: $\left(\frac{409}{77}, \frac{344}{77}, \frac{-212}{77}\right)$ Ans.

19) Reqd Prob = P(1 at an even throw)

= P(1 at 2nd throw or 4th throw or 6th throw or ------)

$= \frac{5}{6}\times\frac{1}{6} + \left(\frac{5}{6}\right)^3\times\frac{1}{6} + \left(\frac{5}{6}\right)^5\times\frac{1}{6} + \cdots\cdots$

$= \frac{1}{6}\left[\underbrace{\frac{5}{6} + \left(\frac{5}{6}\right)^3 + \left(\frac{5}{6}\right)^5 + \cdots\cdots}_{\text{is an infinite G.P with 1st term} = 5/6 \text{ \& common ratio} = \left(\frac{5}{6}\right)^2}\right]$

$\therefore$ Regd Prob $= \frac{1}{6}\left[\frac{5/6}{1-(\frac{5}{6})^2}\right]$ $\left(\because S_\infty = \frac{a}{1-r}\right)$

$= \frac{1}{6} \times \frac{5}{6} \times \frac{36}{11}$

$= \frac{5}{11}$ Ans.

OR

Here $S = \{00, 01, 02, 03, \ldots\ldots 49\}$

A → product of digits on selected ticket is zero $= \{00, 01, 02, 03, 04, 05, 06, 07, 08, 09, 10, 20, 30, 40\}$

B → sum of digits on selected ticket is 8.

$= \{08, 17, 26, 35, 44\}$ $\therefore A \cap B = \{08\}$.

Regd Prob $= P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{1/50}{14/50}$

$= 1/14$ Ans.

Section-C

20) $f: R \to R$ s.t. $f(x) = 2x - 3$

& $g: R \to R$ s.t. $g(x) = x^3 + 5$

$\therefore fog(x) = f(g(x)) = f(x^3+5) = 2(x^3+5) - 3$

$fog(x) = 2x^3 + 7$

$\therefore fog: R \to R$ s.t. $fog(x) = 2x^3 + 7$.

To check if fog is one-one:

Let $fog(x_1) = fog(x_2)$; $x_1, x_2 \in R_{dom}$.

$\Rightarrow 2x_1^3 + 7 = 2x_2^3 + 7$

$\Rightarrow x_1^3 - x_2^3 = 0$

$\Rightarrow (x_1 - x_2)(x_1^2 + x_2^2 + x_1 x_2) = 0$

$\Rightarrow x_1 = x_2$

$\therefore$ fog is one-one.

To check if fog is onto:

Let $y \in$ Codomain R

$\Rightarrow y = fog(x)$

$\Rightarrow y = 2x^3 + 7$

$\Rightarrow \left(\frac{y-7}{2}\right)^{1/3} = x \in$ dom R.

$\Rightarrow$ $\forall$ element in the codomain $\exists$ a pre-image in the domain

$\Rightarrow$ fog is onto.

$\therefore$ fog is one-one, onto $\Rightarrow$ It is a bijection.

$\Rightarrow (fog)^{-1}$ exists & is defined as:

$(fog)^{-1} : R \longrightarrow R$

s.t. - $(fog)^{-1}(y) = \left(\frac{y-7}{2}\right)^{1/3}$.

OR

Reflexivity: $(a,b)R(a,b) \because a+b = b+a \quad \forall\ a,b \in A$

$\therefore$ R is reflexive

Symmetry: Let $(a,b)R(c,d)$

$\Rightarrow a+d = b+c$

$\Rightarrow b+c = a+d$

$\Rightarrow c+b = d+a$

$\Rightarrow (c,d)R(a,b)$ $\therefore$ R is symmetric.

Transitivity:

Let $(a,b)R(c,d)$ & $(c,d)R(e,f)$

$\Rightarrow a+d = b+c$ & $c+f = d+e$

$\therefore a+d+c+f = b+c+d+e$

$\Rightarrow a+f = b+e$

$\Rightarrow (a,b)R(e,f)$ $\Rightarrow$ R is transitive.

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$\because R$ is reflexive, symmetric & transitive

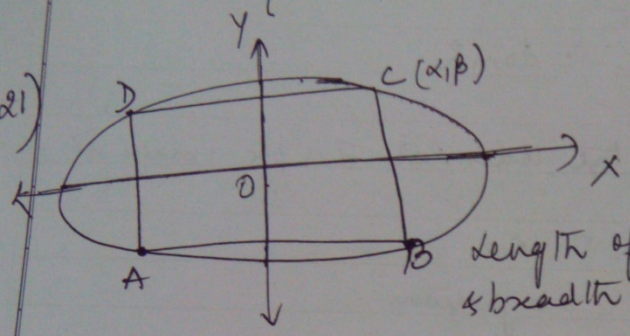
$\Rightarrow R$ is an equivalence relation

Equivalence class $[(2,5)]$ is :

$\{(2,5), (1,4), (3,6), (4,7), (5,8), (6,9)\}$ Ans.

Rough
$A \in (x,y)$
s.t
$(2,5) R (x,y)$
i.e $2+y = 5+x$

21)



Length of rectangle ABCD $= 2\alpha$
& breadth $= 2\beta$

$\therefore$ Ar(ABCD) $= 2\alpha \times 2\beta = 4\alpha\beta$.

(α,β) lies on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\Rightarrow \frac{\alpha^2}{a^2} + \frac{\beta^2}{b^2} = 1$

$\Rightarrow \beta^2 = b^2\left(1 - \frac{\alpha^2}{a^2}\right)$

$\Rightarrow \beta = \frac{b}{a}\sqrt{a^2-\alpha^2}$

$\therefore$ Ar(ABCD) $= A = 4\alpha \cdot \frac{b}{a}\sqrt{a^2-\alpha^2}$

or $A = \frac{4b}{a}\alpha\sqrt{a^2-\alpha^2}$ — (1)

$\therefore \frac{dA}{d\alpha} = \frac{4b}{a}\left\{\sqrt{a^2-\alpha^2} + \alpha \cdot \frac{1}{2\sqrt{a^2-\alpha^2}}(-2\alpha)\right\}$

$= \frac{4b}{a}\left\{\frac{a^2-\alpha^2-\alpha^2}{\sqrt{a^2-\alpha^2}}\right\} = \frac{4b(a^2-2\alpha^2)}{a\sqrt{a^2-\alpha^2}}$

For max^m area, $\frac{dA}{d\alpha} = 0$

$\Rightarrow a^2 - 2\alpha^2 = 0$

$\Rightarrow a^2 = 2\alpha^2$

$\Rightarrow \alpha = \frac{a}{\sqrt{2}}$ $\quad \therefore \beta = \frac{b}{a}\sqrt{a^2 - \frac{a^2}{2}} = \frac{b}{a} \times \frac{a}{\sqrt{2}}$

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$$\frac{d^2A}{d\alpha^2} = \frac{4b}{a}\left\{\frac{\sqrt{a^2-\alpha^2}\,(-4\alpha) - \frac{(a^2-2\alpha^2)}{2\sqrt{a^2-\alpha^2}}(-2\alpha)}{a^2-\alpha^2}\right\}$$

$$= \frac{4b}{a(a^2-\alpha^2)}\left\{\frac{-4\alpha(a^2-\alpha^2) + \alpha(a^2-2\alpha^2)}{\sqrt{a^2-\alpha^2}}\right\}$$

When $\alpha = \frac{a}{\sqrt{2}}$ or $2\alpha^2 = a^2$; $\frac{d^2A}{d\alpha^2} < 0$

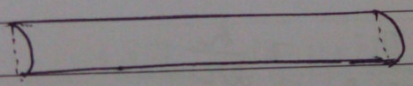
$\Rightarrow$ A is greatest when $\alpha = \frac{a}{\sqrt{2}}$.

& the greatest area $= \frac{4b}{a} \times \frac{a}{\sqrt{2}}\sqrt{a^2 - \frac{a^2}{2}}$

$$= \frac{2\cancel{4}b}{\cancel{\sqrt{2}}} \times \frac{a}{\cancel{\sqrt{2}}}$$

$\Rightarrow$ Regd greatest area $= 2ab$ equnits Ans.

OR



Volume of half cylinder, $V = \frac{\pi r^2 h}{2}$ — (1) (given)

$$S = \underbrace{\frac{\pi r^2}{2} + \frac{\pi r^2}{2}}_{\text{ar. of semi circular ends}} + \underbrace{\frac{1}{2} \times 2\pi r h}_{\text{curved surface area}} + \underbrace{h \times 2r}_{\text{area of the rectangular base}}$$

(S: Total surface area)

$\Rightarrow S = \pi r^2 + \pi r h + 2rh$

$\Rightarrow S = \pi r^2 + (\pi + 2) rh$

$\Rightarrow S = \pi r^2 + (\pi+2)\cancel{r}\cdot\frac{2V}{\pi\cancel{r}^2}$ (using (1))

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$\Rightarrow S = \pi r^2 + (\pi+2)\frac{2V}{\pi r}$ — (2)

$\therefore \frac{dS}{dr} = 2\pi r + \frac{2V}{\pi}(\pi+2)\left(\frac{-1}{r^2}\right)$

for minimum S, $\frac{dS}{dr} = 0 \Rightarrow 2\pi r = \frac{2V}{\pi r^2}(\pi+2)$

$\Rightarrow r^3 = \frac{V}{\pi^2}(\pi+2)$

$\therefore \frac{d^2S}{dr^2} = 2\pi + \frac{2V}{\pi}(\pi+2)\left(\frac{2}{r^3}\right) > 0$ for $r = \left\{\frac{V(\pi+2)}{\pi^2}\right\}^{1/3}$.

$\Rightarrow$ S is minimum when $r = \left\{\frac{V(\pi+2)}{\pi^2}\right\}^{1/3}$

when S is min^m, $r^3 = \frac{V}{\pi^2}(\pi+2)$

$\Rightarrow \pi^2 r^3 = \frac{\pi r^2 h}{2}(\pi+2)$

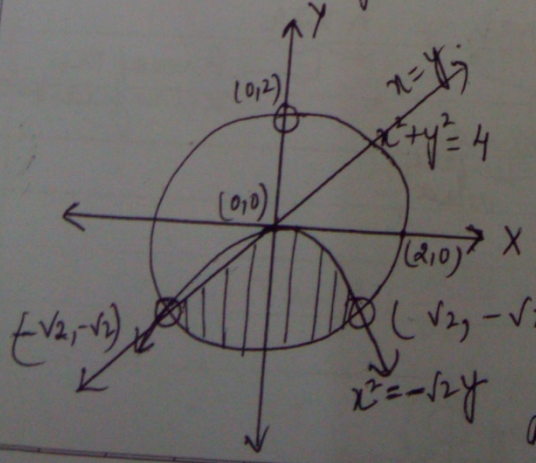
$\Rightarrow \frac{2\pi^2 r^3}{\pi r^2} = h(\pi+2)$

$\Rightarrow 2\pi r = h(\pi+2)$

$\Rightarrow \frac{\pi}{\pi+2} = \frac{h}{2r}$

$\Rightarrow$ height : diameter = $\pi : (\pi+2)$ Hence Proved.

22)



$x^2+y^2 = 4$ — (1)

$x^2 = -\sqrt{2}y$ — (2) (Parabola with vertex (0,0) opening ↓)

$x = y$ — (3)

The shaded part of the figure represents the reqd region &

area of this region $= A = \int_{-\sqrt{2}}^{0} \left(x - \sqrt{4-x^2}\right)dx + \int_{0}^{\sqrt{2}} \left(\frac{-x^2}{\sqrt{2}} - \sqrt{4-x^2}\right)$

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$$\Rightarrow A = \left(\int_{-\sqrt{2}}^{0} x\,dx\right) + \left[\int_{0}^{\sqrt{2}} \left(\frac{-x^2}{\sqrt{2}}\right) dx\right] - \left[\int_{-\sqrt{2}}^{\sqrt{2}} \sqrt{4-x^2}\,dx\right]$$

$$\Rightarrow A = \left[\frac{x^2}{2}\right]_{-\sqrt{2}}^{0} + \frac{1}{\sqrt{2}}\left[\frac{-x^3}{3}\right]_{0}^{\sqrt{2}} - \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_{-\sqrt{2}}^{\sqrt{2}}$$

$$\Rightarrow A = \left[0 - \frac{2}{2}\right] + \frac{1}{3\sqrt{2}}\left[-2\sqrt{2} - 0\right] - \left[\left(\frac{\sqrt{2}}{2}\times\sqrt{2} + \frac{\sqrt{2}}{2}\times\sqrt{2}\right) + 2\left(\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) - \sin^{-1}\left(\frac{-1}{\sqrt{2}}\right)\right)\right]$$

$$\Rightarrow A = -1\ \frac{2}{3} - \left[1+1+2\left(\frac{\pi}{4}+\frac{\pi}{4}\right)\right]$$

$$\Rightarrow A = -1 - \frac{2}{3} - \left[2 + 2\left(\pi/2\right)\right]$$

$$\Rightarrow A = -1 - \frac{2}{3} - \left[2+\pi\right]$$

$$\Rightarrow A = -1 - \frac{2}{3} - 2 - \pi = \frac{-11}{3} - \pi$$

$$\Rightarrow A = \frac{11+3\pi}{3} \text{ sq units Ans.}$$

↓ Reqd Area.

23) $A \longrightarrow$ 1st 13 cards drawn are all red

$E_1 \rightarrow$ missing card is black

$E_2 \rightarrow$ " " " red.

$$\text{Reqd Prob} = P(E_1/A) = \frac{P(E_1)\,P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)}$$

(By Bayes Theorem)

$$\Rightarrow P(E_1/A) = \frac{{}^{26}C_1 \times {}^{26}C_{13}}{{}^{52}C_1} \Big/ \left(\frac{{}^{26}C_1}{{}^{52}C_1} \times \frac{{}^{26}C_{13}}{{}^{51}C_{13}} + \frac{{}^{26}C_1}{{}^{52}C_1} \times \frac{{}^{25}C_{13}}{{}^{51}C_{13}} \right)$$

$$= \frac{\frac{26}{52} \times \frac{{}^{26}C_{13}}{{}^{51}C_{13}}}{\frac{26}{52} \times \frac{{}^{26}C_{13}}{{}^{51}C_{13}} + \frac{26}{52} \times \frac{{}^{25}C_{13}}{{}^{51}C_{13}}}$$

$$= \frac{\frac{26!}{13!\,13!}}{\frac{26!}{13!\,13!} + \frac{25!}{12!\,13!}}$$

$$= \frac{26 \times 25!}{13 \times 12!} \Big/ \left(\frac{26 \times 25!}{13 \times 12!} + \frac{25!}{12!} \right) = \frac{2}{2+1}$$

$= 2/3$ Ans.

24) Eq of plane passing through the point $(-2,-2,2)$ is:

$$A(x+2) + B(y+2) + C(z-2) = 0 \quad -(1)$$

Eq of line joining the points $(1,1,1)$ & $(1,-1,2)$ is:

$$\frac{x-1}{0} = \frac{y-1}{-2} = \frac{z-1}{1} \quad -(2)$$

(1) contains (2) $\Rightarrow A(1+2) + B(1+2) + C(1-2) = 0$

i.e $3A + 3B - C = 0 \quad -(3)$

& $0.A + (-2)B + 1(C) = 0$

i.e $-2B + C = 0 \quad -(4)$

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Solving (3) & (4) we get:

$$\frac{A}{3-2} = \frac{-B}{3-0} = \frac{C}{-6-0} = \lambda$$

$$\Rightarrow \frac{A}{1} = \frac{B}{-3} = \frac{C}{-6} = \lambda \quad \Rightarrow \quad \left.\begin{array}{l} A = \lambda \\ B = -3\lambda \\ \& \; C = -6\lambda \end{array}\right\}$$

Putting values of A, B & C in eq (1) we get:

$\lambda(x+2) - 3\lambda(y+2) - 6\lambda(z-2) = 0$

i.e $x+2-3y-6-6z+12=0$

i.e $x-3y-6z+8=0$

OR $x-3y-6z=-8$

$$\Rightarrow \frac{x}{-8} + \frac{y}{8/3} + \frac{z}{8/6} = 1$$

$$\Rightarrow \frac{x}{-8} + \frac{y}{8/3} + \frac{z}{4/3} = 1$$

∴ Sum of intercepts on co-ordinate axes is $|-8| + |8/3| + |4/3|$

$$= 8 + 8/3 + 4/3$$

$$= \frac{24+8+4}{3} = \frac{36}{3} \text{ 12 units Ans.}$$

26) Let the pt. of contact be (x, y)

∴ Eq of tangent is: $(Y-y) = \left.\frac{dy}{dx}\right]_{(x,y)} (X-x) \quad -(1)$

For intercept on x-axis, $Y=0$

$$\Rightarrow -y = \frac{dy}{dx}(X-x)$$

$$\Rightarrow X = \frac{x\frac{dy}{dx} - y}{dy/dx} = \frac{x - \frac{y}{dy/dx}}{} = x - y\frac{dx}{dy}$$

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P.T.O

(2°)

By given condition, $x - y^d$

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$\Rightarrow X = x - y\frac{dx}{dy}$ — (2)

Acc. to given condition, $x - y\frac{dx}{dy} = y$

$\Rightarrow x - y = y\frac{dx}{dy}$

$\Rightarrow \frac{dx}{dy} = \frac{x-y}{y}$

or $\frac{dx}{dy} - \frac{x}{y} = -1$ which is a L.D.E of the type $\frac{dx}{dy} + Px = Q$,

$I.F = e^{\int \frac{1}{y} dy}$

$\Rightarrow I.F = e^{-\ln y}$

$\Rightarrow I.F = e^{\ln y^{-1}} = 1/y$.

∴ Soln is:

$x\left(\frac{1}{y}\right) = \int -1.\left(\frac{1}{y}\right) dy$

$\Rightarrow \frac{x}{y} = -\ln|y| + \ln C$

$\Rightarrow \frac{x}{y} = \ln\left(\frac{C}{y}\right)$

$\Rightarrow \frac{x}{y} = \log_e\left(\frac{C}{y}\right)$

or $e^{x/y} = C/y$

or $\boxed{ye^{x/y} = C}$ is the reqd. curve Ans.

X ——— X ——— X ——— X ———