Class - XII (Mathematics) classmate Date Page

Questions for Practice during ~~Summer Break~~

Topics covered: • Matrices & Determinants
• Continuity & Differentiability.

Q1) If $\begin{vmatrix} x^k & x^{k+2} & x^{k+3} \\ y^k & y^{k+2} & y^{k+3} \\ z^k & z^{k+2} & z^{k+3} \end{vmatrix} = (x-y)(y-z)(z-x)\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$,

find k using properties of determinants. (Ans: $k=-1$)

Q2) Solve for x using properties of determinants:

$$\begin{vmatrix} x+2 & 2x+3 & 3x+4 \\ 2x+3 & 3x+4 & 4x+5 \\ 3x+5 & 5x+8 & 10x+17 \end{vmatrix} = 0$$

(Ans: $x=-1,-2$)

Q3) Find λ s.t. the following system of equations has no solution:

$2x-y-2z=2$, $x-2y+z=-4$, $x+y+\lambda z=4$.

(Ans: $\lambda=-3$)

Q4) Discuss consistency for the following system of equations. If possible, solve:

(i) $x+4y-2z=3$
$3x+y+5z=7$
$2x+3y+z=5$

(ii) $x+y+z=6$
$x-y+z=2$
$2x+y-z=1$

(Ans (i) No soln (ii) $x=1$, $y=2$, $z=3$)

Q5) Evaluate using properties of determinants:

$$\begin{vmatrix} x+1 & x+2 & x+4 \\ x+3 & x+5 & x+8 \\ x+7 & x+10 & x+14 \end{vmatrix}$$

(Ans: -2)

Q6) If $\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} A = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$, find A

Ans: $A = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$

Q7) Find λ s.t. $\begin{bmatrix} 2 & 0 & 7 \\ 0 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} -\lambda & 14\lambda & 7\lambda \\ 0 & 1 & 6 \\ \lambda & -4\lambda & -2\lambda \end{bmatrix}$ is an identity matrix

(Ans: $\lambda = 1/5$)

Q8) If $2X - Y = \begin{pmatrix} 3 & -3 & 0 \\ 3 & 3 & 2 \end{pmatrix}$ & $2Y + X = \begin{pmatrix} 4 & 1 & 5 \\ -1 & 4 & -4 \end{pmatrix}$, matrices find X & Y.

Ans: $X = \begin{pmatrix} 2 & -1 & 1 \\ 1 & 2 & 0 \end{pmatrix}$ & $Y = \begin{pmatrix} 1 & 1 & 2 \\ -1 & 1 & -2 \end{pmatrix}$

Q9) Find the order of $\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$.

(Ans: 1×1)

Q10) If A and B are two matrices s.t. $AB = B$ and $BA = A$, find $A^2 + B^2$ in terms of A & B

(Ans: $A + B$)

Q11) If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$ whenever $A^2 = B$, find α

(Ans: No real value of α exists)

Q12) For a 3×3 matrix if $|A| = 4$, find $|\text{Adj } A|$.

(Ans: 16).

Q13) If $A = \begin{bmatrix} a \\ b \\ -a \end{bmatrix} [a \quad b \quad -a]$, find if A^{-1} exists.

(Ans: No)

Q14) Classify the type of matrix: $\begin{bmatrix} 0 & 5 & -7 \\ -5 & 0 & 11 \\ 7 & -11 & 0 \end{bmatrix}$

Q15) If $A = \begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix}$, find $A'A$ where A' is transpose of A.

(Ans: $-I$)

Q16) If A & B are symmetric matrices, is $AB - BA$ symmetric or skew symmetric? (Ans: Skew Symm

Q17) Find a, b if $\begin{pmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{pmatrix} \begin{pmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{pmatrix}^{-1} = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$

(Ans: $a = \cos 2\theta$, $b = \sin 2\theta$)

Q18) Suppose a matrix A satisfies $A^2 - 5A + 7I = 0$. If $A^8 = aA + bI$, find a (Ans: 1265)

Q19) If $\begin{bmatrix} i & 0 \\ 3 & -i \end{bmatrix} + X = \begin{bmatrix} i & 2 \\ 3 & 4+i \end{bmatrix} - X$, find X.

Ans: $X = \begin{bmatrix} 0 & 1 \\ 0 & 2+i \end{bmatrix}$

Q20) Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 5 \end{bmatrix}$. If $A^{-1} = xA + yI$, find $x + 2y$.

(Ans: 1)

Q21) Prove using properties of determinants:

$$\begin{vmatrix} 1+x & y & y \\ 1 & x+y & 1 \\ 1 & x & 1+y \end{vmatrix} = 4xy$$

Q22) If $x = \sec\theta - \cos\theta$, $y = \sec^n\theta - \cos^n\theta$, show that:

$$(x^2+4)\left(\frac{dy}{dx}\right)^2 = n^2(y^2+4)$$

Q23) If $y = \sin^{-1}\sqrt{\frac{1-x}{1+x}}$, find the value of $\frac{-3}{\sqrt{2}}\frac{dy}{dx}\Big]_{x=1/2}$

(Ans: 2)

Q24) If $f(x) = \begin{cases} ax+b, & x \le -1 \\ ax^3+x+2b, & x > -1 \end{cases}$ is differentiable for all values of x, find a, b

(Ans: $a = -1/2$, $b = 1$)

Q25) Find the derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w.r.t. $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x=0$ (Ans: $1/4$)

Q26) Check continuity & differentiability for the function $f(x) = \begin{cases} x^4+x^2-x+2, & x \le 1 \\ 3x^3-x^2+x, & x > 1 \end{cases}$ at $x=1$.

(Ans: continuous at $x=1$ but Not differentiable at $x=1$)

Q27) If $x = \sin^{-1}t$ & $y = \log(1-t^2)$, find $\frac{d^2y}{dx^2}$ at $t = 1/2$

(Ans: $-8/3$)

Q28) If $f(1) = 3$ & $f'(1) = -1/3$, find the derivative of $(x^{11} + f(x))^{-2}$ at $x = 1$ (Ans: $-1/3$)

Q29) If $(a+bx)e^{y/x} = x$, P.T. $x^3 \frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$.

Q30) If $x = \frac{1+t}{t^3}$, $y = \frac{3}{2t^2} + \frac{2}{t}$, P.T.

$$x\left(\frac{dy}{dx}\right)^3 = 1 + \frac{dy}{dx}$$

Q31) If $y = \sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right) + \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

P.T. $\frac{dy}{dx} = \frac{-1}{2(1+x^2)}$

Q32) If $\tan y = \frac{2t}{1-t^2}$ and $\sin x = \frac{2t}{1+t^2}$, P.T. $\frac{dy}{dx} = 1$

Q33) Differentiate $\frac{\tan^{-1}x}{1+\tan^{-1}x}$ w.r.t. $\tan^{-1}x$.

(Ans: $\frac{1}{(1+\tan^{-1}x)^2}$)

Q34) Differentiate $x^{\sin^{-1}x}$ w.r.t. $\sin^{-1}x$.

Ans: $x^{\sin^{-1}x}\left[\log x + (\sin^{-1}x)\frac{\sqrt{1-x^2}}{x}\right]$.

Q35) If $x^p y^q = (x+y)^{p+q}$, P.T. $\frac{d^2y}{dx^2} = 0$

Q36) If $p^2 = a^2\cos^2\theta + b^2\sin^2\theta$ P.T:

$$p + \frac{d^2p}{d\theta^2} = \frac{a^2b^2}{p^3}$$

Q37) If $xe^{xy} = y + \sin^2 x$ find $\left.\frac{dy}{dx}\right]_{x=0}$. (Ans: 1)

Q38) If $y = \tan^{-1}\sqrt{x^2-1}$ P.T:

$$x(x^2-1)y_2 + (2x^2-1)y_1 = 0$$

Q39) If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$ P.T.

$$x^2y_2 + xy_1 + n^2y = 0$$

Q40) If $y = (\sin^{-1}x)^2 + (\cos^{-1}x)^2$ P.T.

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 4.$$

Q41) Find inverse of the matrix $A = \begin{bmatrix} 2 & -1 & 4 \\ 4 & 0 & 2 \\ 3 & -2 & 7 \end{bmatrix}$ using elementary row/column transformations.

Ans: $\begin{bmatrix} -2 & 1/2 & 1 \\ 11 & -1 & -6 \\ 4 & -1/2 & -2 \end{bmatrix}$

Q42) A mixture is to be made of 3 types of food A, B & C containing proteins, vitamins and minerals P, Q, R respectively as shown below:

Food	P (No. of units)	Q	R
A	1	2	5
B	3	1	1
C	4	2	1

How many kgs of food A, B, C be mixed s.t. The mixture contains 8 units of P, 5 units of Q and 7 units of R? Would you recommend this mixture as a part of our daily diet? Justify your answer. (Ans: 1 kg each of food A, B & C)

Q43) If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ are two square

matrices, find AB and hence solve the given system of linear equations:

$$x - y = 3$$
$$2x + 3y + 4z = 17$$
$$y + 2z = 7$$

(Ans: $x = 2, y = -1, z = 4$)

Q44) Discuss the continuity of the given function:

$$f(x) = |x-1| + |x+1|$$

Also discuss differentiability at $x = 1$.

(Ans: continuous everywhere. Is not differentiable at $x = 1$)

Q45) Find a, b if:

$$f(x) = \begin{cases} \dfrac{1-\sin^3 x}{3\cos^2 x}, & x < \pi/2 \\ a, & x = \pi/2 \\ \dfrac{b(1-\sin x)}{(\pi - 2x)^2}, & x > \pi/2 \end{cases}$$

is continuous at $x = \pi/2$.

(Ans: $a = 1/2, \quad b = 4$)

Q46) Find a, b if:

$$f(x) = \begin{cases} x^2/a, & 0 \le x < 1 \\ a, & 1 \le x < \sqrt{2} \\ \dfrac{2b^2 - 4b}{x^2}, & \sqrt{2} \le x < \infty \end{cases}$$

is continuous on $[0, \infty)$

(Ans: $a = -1, b = 1$ or $a = 1, b = 1 \pm \sqrt{2}$)

Q47) If $f(x) = \begin{cases} ax^2 + b, & 0 \le x < 1 \\ 4, & x = 1 \\ x + 3, & 1 < x \le 2 \end{cases}$ then the value of (a, b) for which $f(x)$ is not continuous at $x = 1$ is:

(a) $(2, 2)$ (b) $(3, 1)$ (c) $(4, 0)$ (d) $(5, 2)$

(Ans: (d))

Q48) If $y = x + e^x$, find $\frac{d^2x}{dy^2}$. (Ans: $\frac{-e^x}{(1+e^x)^3}$)

Q49) If $x = \cos\theta$, $y = \sin^3\theta$; Prove that:

$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 3\sin^2\theta\left(5\cos^2\theta - 1\right)$$

Q50) If $x = \sin t$, $y = \sin pt$; Prove that:

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$$