HOLIDAY HOMEWORK (SUMMER BREAK)

CLASS XI - 2015-16

SUB : MATHEMATICS QUESTION BANK

CHAPTERS COVERED: 1. Sets 2. Relation & Functions 3. Trigonometry

1. Write the set of all positive integers whose cube is odd.
2. Describe the following set in Roster form:
   a) {x : x is a prime no. which is a divisor of 60}
   b) {x : x is a two digit no. such that the sum of its digits is 7}
3. Write the set in set builder form:

$$\left\{\frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \frac{4}{17}, \frac{5}{26}, \frac{6}{37}, \frac{7}{50}\right\}$$

4. Which of the following sets are empty sets?
   1. $D = \{x : x^2 = 25$, & x is an odd integer$\}$
   2. $E = \{x : x^2 - 2 = 0$ & x is rational$\}$
   3. $A = \{x : x$ is a point common to any two parallel lines$\}$
5. Which of the following sets are equal?

   $A = \{x : x \in N, x < 3\}$, $B = \{1, 2\}$, $C = \{3, 1\}$,

   $D = \{x : x \in N, x$ is odd, $x < 5\}$, $E = \{1, 2, 1, 1\}$

   $F = \{1, 1, 3\}$, $E = \{1, 2, 1, 1\}$
6. Consider the following sets: ϕ, $A = \{1, 2\}$,

   $B = \{1, 4, 8\}$, $C = \{1, 2, 4, 6, 8\}$

   Insert the correct symbol $\subset$ or $\not\subset$ between each of the following pair of sets:

   i) ϕ __ B ii) A __ B iii) A __ C

   iv) B __ C
7. Describe among the following sets, which are subsets of which:

$A = \{x : x \text{ satisfies } x^2 - 8x + 12 = 0\}$, $B = \{2, 4, 6\}$

$C = \{2, 4, 6, 8, \ldots\}$, $D = \{6\}$.

8. For sets A, B & C, using property of sets, prove that

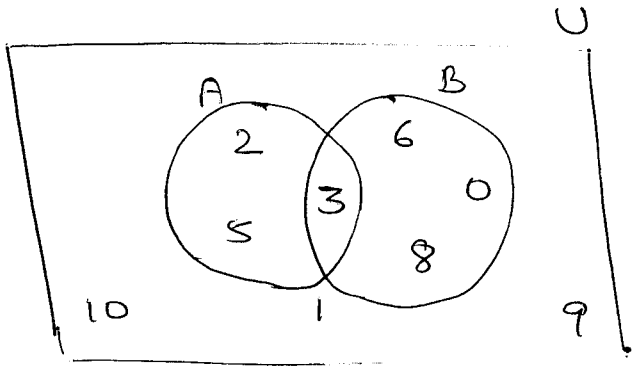
i) $A - (B - C) = (A - B) \cup (A \cap C)$

ii) $(A \cup B) - C = (A - C) \cup (B - C)$

iii) $(A - B) \cap A = A \cap B'$

iv) $A - (B \cup C) = (A - B) \cap (A - C)$

9. From the adjoining Venn diagram, determine the following sets.



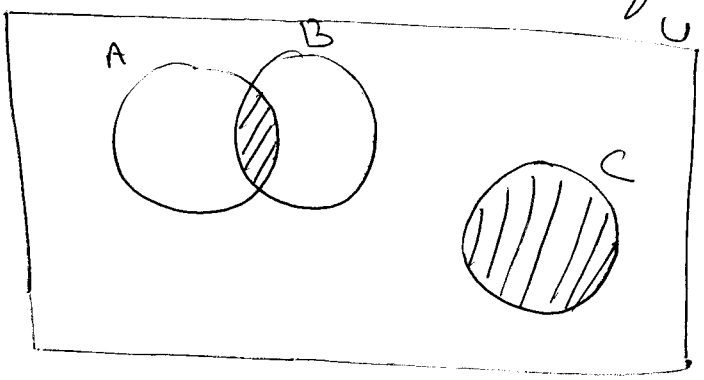
i) $A \cup B$

ii) $A \cap B$

iii) $A - B$

iv) $(A \cap B)'$

10. Express the shaded region of the Venn diagram in terms of union & intersection of the sets, A, B & C



Also, if $n(U) = 50$, $n(A) = 26$, $n(C) = 12$, $n(A \cap B) = 2$ & $n(A' \cap B') = 27$, find $n(B)$

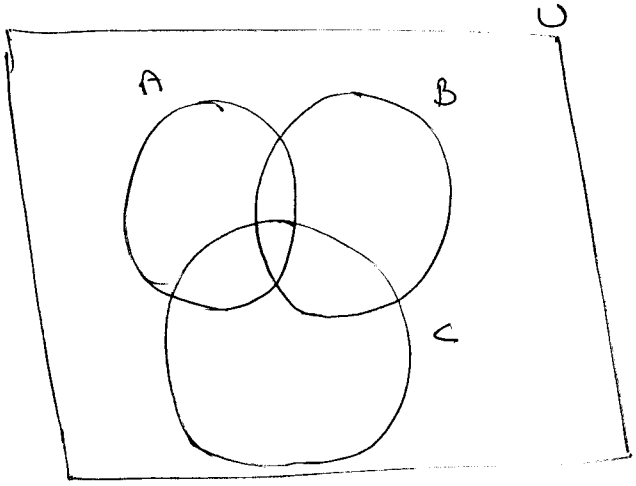
11. Let $U=\{x \in N : x \leq 8\}$, $A=\{x \in N : 5 < x^2 < 50\}$ & $B=\{x \in N ; x$ is a prime no. less than $10\}$

Draw a Venn diagram to show the relationship between the given sets. Hence list the elements of the following sets i) A' ii) B' iii) $(A-B)$ iv) $A \cap B'$ v) Is $A-B = A \cap B'$?

12. Two finite sets have m & n elements. The total no. of subsets of the first set is 56 more than the total no. of subsets of second set. Find the values m & n

13. Shade the following sets on the Venn diagram:

i) $A' \cap (B \cup C)$ ii) $A' \cap (C-B)$



14. Classify the following as a finite or infinite set:

i) $\{x \in N : (x-1)(x-2)\} = 0$ ii) $\{x \in N : x^2 \geq 9\}$

iii) $\{x \in N : x$ is odd$\}$

15. In a survey of 100 persons it was found that 28 read magazine A, 30 read magazine B, 42 read magazine C, 8 read magazines A & B, 10 read magazines A & C, 5 read magazines B & C & 3 read all the three magazines. Find:

i) How many read none of three magazines?

ii) How many read magazine C only?

16. A survey was conducted on the T.V programmes watched by 120 students of a school hostel. It was revealed that 70 students watched 'Discovery Channel' & 56 students watched 'Sports channel' where as 24 watched both the programmes. Find the no. of students who did not watch T.V on that day.

17. In a survey conducted on a group of 2000 people, it is found that 1450 liked product A & 1720 people liked product B. What is the least no. of people who liked both the products A & B?

18. Find x & y : if $(x+3, 5) = (6, 2x+y)$

19. Let $f: R \to R$ be given by $f(x) = x^2 + 3$

Find i) $\{x : f(x) = 28\}$ ii) The pre-images of 39 & 2 under f.

20. Determine the domain & range of the relation R defined by

$R = \{(x+1, x+5) : x \in \{0, 1, 2, 3, 4, 5\}\}$

21. If $A \times B = \{(a,1), (a,2), (a,5), (b,2), (b,5), (b,1)\}$ find $B \times A$.

22. Let A & B be two sets such that $A \times B$ consist of 6 elements. If three elements of $A \times B$ are $(1,4)$, $(2,6)$, $(3,6)$ find i) $A \times B$

23. Given $A=\{-1, 0, 2, 5, 6, 11\}$, $B=\{-2, -1, 0, 1, 2, 8, 10\}$ & $f(x)=x^2-x-2$. Find $f(A)$. Is $f(A)=B$?

24. Is $g=\{(1,1),(2,3),(3,5),(4,7)\}$ a function? If this is described by formula $g(x)=\alpha x+\beta$, then what values should be assigned to α & β

25. Let $f: R \to R$ be such that $f(x)=2^x$. Determine
i) $\{x : f(x)=1\}$ ii) whether $f(x+y)=f(x)\cdot f(y)$ holds

26. If $A=\{a, b, c\}$; $B=\{d\}$, $C=\{e\}$. verify that $A\times(B\cup C)=(A\times B)\cup(A\times C)$.

27. Find the domain of the following functions:

i) $f(x)=\dfrac{x}{x^2+3x+2}$ ii) $f(x)=\dfrac{x^3-x+3}{x^2-1}$

iii) $f(x)=x/|x|$ iv) $f(x)=\dfrac{3x}{2x-8}$

v) $f(x)=\dfrac{1}{\sqrt{x^2-1}}$ vi) $f(x)=\dfrac{1}{1-x^2}$

28. ~~Def~~ Find the range of the following functions

i) $f(x)=\dfrac{3}{2-x^2}$ ii) $f(x)=1-|x-2|$

iii) $f(x)=\sqrt{16-x^2}$ iv) $f(x)=\dfrac{|x-4|}{x-4}$

29. Draw the graph of

$$f(x)=\begin{cases} 3-x, & x>1 \\ 1, & x=1 \\ 2x, & x<1 \end{cases}$$

30. Let $f(x)=g(x)=\sqrt{x}$ & $g(x)=x$ be two functions, defined in the domain $R^+\cup\{0\}$. Find

i) $(f+g)(x)$ ii) $(f-g)(x)$ iii) $(fg)(x)$ iv) $\left(\frac{f}{g}\right)(x)$

31. Express each of the following in sexagesimal system

i) $\frac{\pi}{5}$ ii) $\frac{\pi}{12}$ iii) $\frac{\pi}{8}$ iv) $\frac{1}{4}$ v) -3

32. Find the angle in degrees subtended at the centre of a circle by an arc whose length is 2.2 times the radius

33. Express each of the following angles in radians:

i) 300° ii) $5^\circ 37' 30''$ iii) $-32^\circ 30'$ iv) 225° v) 450°

34. A train is travelling on a curve of 700 m radius at 144 km/hr. Through what angle will it turn in one minute.

35. Find the angle between the minute hand & the hour hand of a clock when the time is 7:20

36. The difference between two acute angles of a right angled triangle is $\frac{3\pi}{10}$ radians. Express the angles in degrees

37. Find the value of

i) $\tan(-1125^\circ)$ ii) $\cos(660^\circ)$ iii) $\sin(-330^\circ)$

iv) $\cot 225^\circ$ v) ~~$\tan(\quad)$~~ $\cos 510^\circ$

38. Prove that

i) $\tan 225^\circ \cot 405^\circ + \tan 765^\circ \cot 675^\circ = 0$

ii) $\tan\frac{11\pi}{3} - 2\sin\frac{2\pi}{3} - \frac{3}{4}\sec^2\frac{\pi}{4} + 4\cot^2\frac{17\pi}{6} = \frac{3-4\sqrt{3}}{2}$

39. If $\sin A = \frac{4}{5}$, & $\cos B = \frac{-5}{13}$, where $0 < A < \pi/2$, $\pi/2 < B < \pi$,

find the value of $\cos(A-B)$

40. Find the value of

i) $\sin 32^\circ \cos 28^\circ + \cos 32^\circ \sin 28^\circ$

ii) $\cos 25^\circ 35' \cos 34^\circ 25^\circ - \sin 25^\circ 35' \sin 34^\circ 25'$

iii) $\dfrac{\tan 69^\circ + \tan 66^\circ}{1 - \tan 69^\circ \tan 66^\circ}$

41. Simplify.

i) $\dfrac{\sin(180+\theta)\cos(360-\theta)\tan(270-\theta)}{\sec^2(90+\theta)\tan(-\theta)\sin(270+\theta)}$

ii) $\dfrac{\cos(2\pi+\theta)\operatorname{cosec}(2\pi+\theta)\tan(\pi/2+\theta)}{\sec(\pi/2+\theta)\cos\theta\cot(\pi+\theta)}$

42. Prove that

i) $\sin^2\left(\frac{\pi}{8}+\frac{A}{2}\right) - \sin^2\left(\frac{\pi}{8}-\frac{A}{2}\right) = \frac{1}{\sqrt{2}}\sin A$

ii) $\tan 13A - \tan 7A - \tan 6A = \tan 13A \tan 7A \tan 6A$.

43. Draw the graph of

i) $\sin x$, $0 \le x \le 2\pi$

ii) $\tan x$, $0 < x \le \pi$

44. If $\tan(A+B) = m$ & $\tan(A-B) = n$, show that

i) $\tan 2A = \dfrac{m+n}{1-mn}$ ii) $\tan 2B = \dfrac{m-n}{1+mn}$

45. Prove each of the following:

i) $\frac{\sin\alpha - \sin\beta}{\cos\beta - \cos\alpha} = \cot\left(\frac{\alpha+\beta}{2}\right)$

ii) $\frac{\sin 75^\circ - \sin 15^\circ}{\cos 75^\circ + \cos 15^\circ} = \frac{1}{\sqrt{3}}$

iii) $\sin\left(\frac{\pi}{4}+A\right)\cdot\sin\left(\frac{\pi}{4}-A\right) = \frac{1}{2}\cos 2A$

iv) $\cos A + \cos(120^\circ - A) + \cos(120^\circ + A) = 0$

v) $(\cos\alpha + \cos\beta)^2 + (\sin\alpha + \sin\beta)^2 = 4\cos^2\frac{\alpha-\beta}{2}$

vi) $2\cos\left(\frac{\pi}{13}\right)\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13} = 0$

vii) $\frac{\sin A + \sin 3A}{\cos A + \cos 3A} = \tan 2A$

viii) $2\cos\theta = \sqrt{2+\sqrt{2+2\cos 4\theta}}$

ix) $\frac{\sec 8\theta - 1}{\sec 4\theta - 1} = \frac{\tan 8\theta}{\tan 2\theta}$

x) $2\sin\left(\frac{\pi}{4}+\alpha\right)\cos\left(\frac{\pi}{4}+\beta\right) = \cos(\alpha+\beta) + \sin(\alpha-\beta)$

46. If $\sin\theta + \sin\phi = a$ & $\cos\theta + \cos\phi = b$, find the values of $\tan\left(\frac{\theta+\phi}{2}\right)$, $\tan\left(\frac{\theta-\phi}{2}\right)$.

47. If $\tan\theta\phi + \sin\theta = m$ & $\tan\theta - \sin\theta = n$, show that $m^2 - n^2 = 4\sqrt{mn}$.

48. If $\sec A = x + \frac{1}{4x}$, then prove that $\sec A + \tan A = 2x$ or $\frac{1}{2x}$

49. Express each as the product

i) $\sin 32^\circ + \sin 54^\circ$ ii) $\cos 31^\circ - \cos 71^\circ$ iii) $\cos 6\theta + \cos 4\theta$

iv) $\cos\theta - \cos\frac{7\theta}{2}$.

ANSWER KEY

1. $\{2n+1, n \in Z, n > 0\}$

2. b) $\{16, 25, 34, 43, 52, 61, 70\}$ a) $\{2, 3, 5\}$

3. $\left\{\frac{n}{n^2+1} ; n \in N, n \leq 7\right\}$

4. 1. D is non-empty set.
   2. E is empty set
   3. A empty set

5. Equal sets : $B = D$, $C = F$

   Equivalent sets : A, E, H ; B, D, G ; C, F

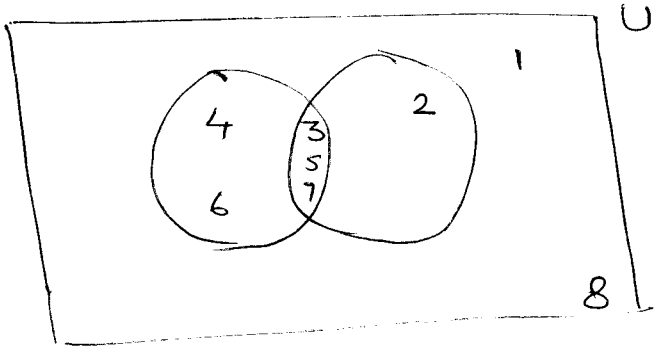
6. i) $\phi \subset B$ ii) $A \not\subset B$ iii) $B \subset C$ iv) $B \subset C$

7. $D \subset A \subset B \subset C$

9. i) $\{0, 2, 3, 5, 6, 8\}$ ii) $\{3\}$ iii) $\{2, 5\}$,
   iv) $\{0, 1, 2, 5, 6, 8, 9, 10, 12\}$

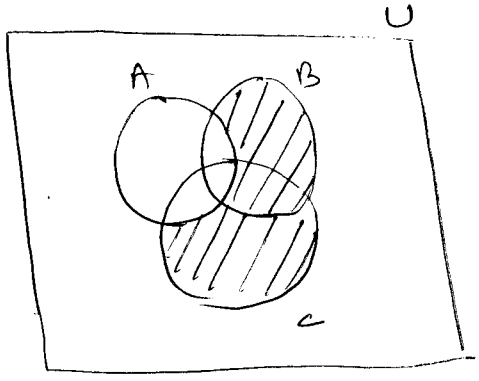
10. i) $(A \cap B) \cup C$ (ii) 17

11. i) $\{1, 2, 8\}$
    ii) $\{1, 4, 6, 8\}$
    iii) $\{4, 6\}$
    iv) $\{4, 6\}$
    v) Yes

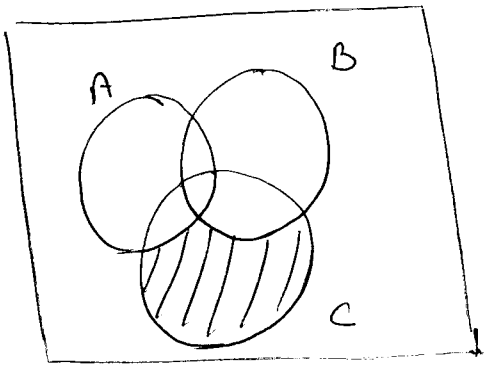


12. $n = 3$ & $m = 6$

13. i)



ii)



14. i) Finite ii) finite iii) infinite

15. i) 20 ii) 30

16. 18.

17. 1170

18. $x = 3, y = -1$

19. i) $\{-5, 5\}$ ii) ± 6 ; does not exist

20. Domain = $\{1, 2, 3, 4, 5, 6\}$

Range = $\{5, 6, 7, 8, 9, 10\}$

21. $B \times A = \{(1,a), (2,a), (5,a), (1,b), (2,b), (5,b)$

22. i) $A \times B = \{(1,4), (2,4), (3,4), (1,6), (2,6), (3$

ii) $B \times A = \{(4,1), (4,2), (4,3), (6,1), (6,2), (6,$

23. i) $f(A) = \{0, -2, 18, 28, 108\}$

& $f(A) \neq B$.

24. yes, $\alpha = 2, \beta = -1$

25. i) $\{0\}$, ii) yes

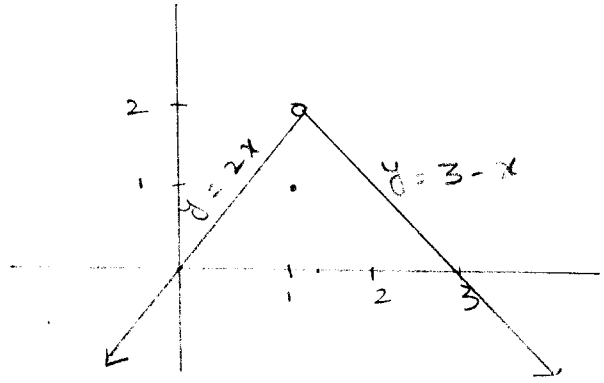
27. i) $R - \{-1, -2\}$ ii) $R - \{1, -1\}$ iii) R

iv) $R - \{4\}$ v) $(-\infty, -1) \cup (1, \infty)$ vi) $x \neq \pm 1$

28. i) $(-\infty, 0] \cup [\frac{3}{2}, \infty)$ ii) $(-\infty, 1)$ iii) $(-4, 4)$

iv) $\{-1, 1\}$

29.



30. i) $\sqrt{x} + x$ ii) $\sqrt{x} - x$ iii) $x^{3/2}$ iv) $x^{1/2}$ $(x \neq 0)$

31. i) 36° ii) 15° iii) $22^\circ 30'$ iv) $14^\circ 19' 5.4''$

32. 126°

33. i) $\frac{5\pi}{8}$ ii) $\frac{\pi}{32}$ iii) $-\frac{13\pi}{72}$ iv) $\frac{5\pi}{4}$ v) $5\pi/2$

34. $\frac{1}{3}$. 35) 100° 36) $72^\circ, 18^\circ$

37. i) -1 ii) $\frac{-2}{\sqrt{3}}$ iii) $\frac{1}{2}$ iv) 1 v) $-\frac{\sqrt{3}}{2}$.

39. $\frac{33}{65}$ 40. i) $\frac{\sqrt{3}}{2}$ ii) $\frac{1}{2}$ iii) -1

41. i) $-\sin\theta \cos^2\theta$

ii) -1