

Maths Holiday Homework

Class - IX

Ch-1 Number Systems

i) Rationalisation

1. If $x = \frac{\sqrt{p+2q} + \sqrt{p-2q}}{\sqrt{p+2q} - \sqrt{p-2q}}$, then show that

$$qx^2 - px + q = 0.$$

2. If $x = \frac{1}{2-\sqrt{3}}$, find the value of $x^3 - 2x^2 - 7x + 5$. [Ans. 3]

3. If $x = 7 - 4\sqrt{3}$, find $\sqrt{x} + \frac{1}{\sqrt{x}}$. [Ans. 4]

4. If $\frac{\sqrt{7}-1}{\sqrt{7}+1} - \frac{\sqrt{7}+1}{\sqrt{7}-1} = a + b\sqrt{7}$, find a and b where a & b are rational. [Ans. a=0, b=-2/3]

5. If $x = \frac{\sqrt{2}+1}{\sqrt{2}-1}$ and $y = \frac{\sqrt{2}-1}{\sqrt{2}+1}$, find the value of $x^2 + y^2 + xy$. [Ans. 35]

6. Prove that $\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2} = 5$

7. Evaluate $\sqrt{5+2\sqrt{6}}$. [Ans. $\sqrt{3}+\sqrt{2}$]

8. Which is smaller out of $\sqrt[3]{10}$ and $\sqrt[3]{9}$. [Ans. $\sqrt[3]{10}$]

9. Evaluate $\sqrt{8-2\sqrt{15}}$. [Ans. $\sqrt{5}-\sqrt{3}$]

10. Express with the rational denominator.

$$\frac{1}{\sqrt{2} + \sqrt{3} + \sqrt{5}} \quad [\text{Ans. } \frac{2\sqrt{3} + 3\sqrt{2} - \sqrt{30}}{12}]$$

11. Find the values of a and b if $\frac{7+3\sqrt{5}}{3+\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}}$
 $= a + \sqrt{5}b$ [Ans. $a=0, b=1$]

12. Express $\frac{1}{\sqrt{3}-\sqrt{2}+1}$ with a rational denominator
[Ans. $\frac{2+\sqrt{6}-\sqrt{2}}{4}$]

13. Prove that $\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} - \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}} - \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} = 1$.

14. Express $\frac{7-3\sqrt{13}}{11-2\sqrt{13}}$ in the form $a+b\sqrt{13}$.
[Ans. $-\frac{1}{69} - \frac{19}{69}\sqrt{13}$]

15. If $x = 7 + \sqrt{40}$, find the value of $\sqrt{x} + \frac{1}{\sqrt{x}}$.
[Ans. $\frac{2}{3}(2\sqrt{5}+2)$]

ii) Exponents of Real Numbers

1. $\frac{\sqrt{(25)^7} \sqrt[3]{128}}{(\sqrt[3]{10^{-1/3}})^6}$ [Simplify] [Ans. 1000]

2. Simplify $\sqrt{\frac{81}{4}} \times \sqrt[3]{\frac{64}{125}} \times \left(\frac{5}{\sqrt{3}}\right)^2$ [Ans. 30]

3. Simplify $\left(\frac{243}{32}\right)^{2/5} \times \left[\left(\frac{9}{25}\right)^{3/2} \div \left(\sqrt[3]{\frac{125}{27}}\right)^{-2}\right]$ [Ans. $\frac{4}{15}$]

4. Prove that $\left(\frac{x^a}{x^b}\right)^c \times \left(\frac{x^b}{x^c}\right)^a \times \left(\frac{x^c}{x^a}\right)^b = 1$.

5. Prove that $\left(\frac{x^{(a+b)^c}}{x^{ab}}\right)^{a-b} \times \left(\frac{x^{(b+c)^a}}{x^{bc}}\right)^{b-c} \times \left(\frac{x^{(c+a)^b}}{x^{ca}}\right)^{c-a} = 1$.

If $2^{2x+5} \times 3^{2x-9} = 3072$, find x [Ans = 5]

7. Solve for x : $5 \times 9^x = 3^{2x+2} - 324$ [Ans $x=2$]

8. If $24^x = \frac{16}{9^y}$, find x & y [Ans $x = \frac{4}{3}$,
 $y = -\frac{2}{3}$]

9. Prove that $\frac{a^{-1}}{a^{-1}+b^{-1}} + \frac{a^{-1}}{a^{-1}-b^{-1}} = \frac{2b^2}{b^2-a^2}$

10. Find the value of x if $5^{x-3} \times 3^{2x-8} = 225$ [Ans $x=5$]

11. If x, y and z are positive real numbers, then show that

$$\sqrt{x^{-1}y} \times \sqrt{y^{-1}z} \times \sqrt{z^{-1}x} = 1$$

12. If $\frac{9^n \times 3^2 \times (3^{-1/2})^{-2} - (27)^n}{3^{3m} \times 2^3} = \frac{1}{27}$, then prove

that $m-n=1$.

13. Simplify $[\{(625)^{-1/2}\}^{-1/4}]^2$ [Ans = $\frac{1}{5}$]

14. Prove that $(\frac{64}{125})^{2/3} + \frac{1}{(\frac{256}{625})^{1/4}} + (\frac{\sqrt{25}}{\sqrt[3]{64}}) = \frac{65}{16}$.

15. Find the value of x if, $2^{x-7} \times 5^{x-4} = 1250$ [Ans $x=8$]

2 Polynomials

Application of Remainder Theorems and Factor Theorems:-

1. If $x = \frac{1}{2}$ is a zero of the polynomial $p(x) = 8x^3 - ax^2 - x + 2$, find the value of a .
[Ans. $a = 6$]
2. Let R_1 and R_2 are the remainders when the polynomials $x^3 + 2x^2 - 5ax - 7$ and $x^3 + ax^2 - 12x + 6$ are divided by $x+1$ and $x-2$. If $2R_1 + R_2 = 6$ find the value of a .
[Ans. $a = 2$]
3. If $f(x) = x^4 - 2x^3 + 3x^2 - ax + b$ is a polynomial such that when it is divided by $x-1$ and $x+1$, the remainders are 5 and 19. Determine the remainder when $f(x)$ is divided by $x-2$.
[Ans. 10]
4. The polynomials $ax^3 + 3x^2 - 13$ and $2x^3 - 5x + a$ are divided by $x+2$. If the remainder in each case is the same, find the value of a .
[Ans. $a = \frac{5}{9}$]
5. Find the integral zeroes of the polynomial $x^3 + 15x^2 + 32x + 20$.
[Ans. -2, -10]
6. Show that $a+4$, $a-3$ and $a-7$ are the factors of $a^3 - 6a^2 - 19a + 84$.
7. Find the values of m and n so that $x-1$ and $x+2$ are factors of the polynomial $x^3 + 10x^2 + mx + n$.
[Ans. $m = 7$
 $n = -18$]

Divide the following, using long division method:-

8. $8x^3 + 1$ by $q(x) = x + 1$

[quotient = $x^2 - x + 1$
remainder = 0]

9. $p(x) = 3x^2 + 2x - 11$, $q(x) = 2x + 1$

[quotient = $\frac{3}{2}x + \frac{1}{4}$
remainder = $-\frac{45}{4}$]

10. $p(x) = x^3 + x - 2$, $q(x) = x - 1$

[quotient = $x^2 + x + 1$
remainder = 0]

11. $p(x) = 4x^3 + 20x^2 + 33x + 18$, $q(x) = 2x + 3$

[quotient = $2x^2 + 7x + 6$
remainder = 0]

Algebraic Identities:-

12. Factorize:- $x^8 - y^8$

[Ans. $(x-y)(x+y)(x^2+y^2)(x^2+y^2-\sqrt{2}xy)(x^2+y^2+\sqrt{2}xy)$]

13. Factorize $x^2 + 3\sqrt{3}x + 6$

[Ans. $(x + 2\sqrt{3})(x + \sqrt{3})$]

14. Factorize $x^4 + 5x^2 + 9$

[Ans. $(x^2 - x + 3)(x^2 + x + 3)$]

15. Factorize $(x - 2y)^3 + (2y - 3z)^3 + (3z - x)^3$

[Ans. $3(x - 2y)(2y - 3z)(3z - x)$]

16. Simplify $\frac{(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3}{(a - b)^3 + (b - c)^3 + (c - a)^3}$

[Ans. $(a + b)(b + c)(c + a)$]

17. If $p = 2 - a$, prove that $a^3 + 6ap + p^3 - 8 = 0$.

18. Find the value of $x^3 - 8y^3 - 36xy - 216$ when $x = 2y + 6$

[Ans. 0]

19. Factorize $7\sqrt{2}x^2 - 10x - 4\sqrt{2}$

[Ans. $(x - \sqrt{2})(7\sqrt{2}x + 4)$]

20. Factorize $x^6 - 7x^3 - 8$

[Ans. $(x - 2)(x + 1)(x^2 + 2x + 4)(x^2 - x + 1)$]

Maths Holiday Home work

Class - IX

Ch-1 Number Systems

1. locate $\sqrt{10.5}$ geometrically on the number line.
2. locate $\sqrt{7}$ on the number line.
3. Find the values of a & b in $\frac{\sqrt{3}-1}{\sqrt{3}+1} = a-b\sqrt{3}$. [Ans: $a=2, b=1$]
4. Rationalise the denominator and simplify.
[Ans: $\frac{5-2\sqrt{6}}{1}$]
- a) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$
- b) $\frac{7+3\sqrt{5}}{3+\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}}$ [Ans: $\sqrt{5}$]
- c) $\frac{1+\sqrt{2}}{3-2\sqrt{2}}$ [Ans: $7+5\sqrt{2}$]
5. Simplify.
a) $(625)^{-\frac{1}{4}}$ [Ans: $\frac{1}{5}$]
b) $(\frac{256}{81})^{\frac{3}{4}}$ [Ans: $\frac{1024}{243}$]
6. Find three rational numbers between $\frac{3}{2}$ and $\frac{5}{3}$.
7. Find four rational numbers between $\frac{5}{2}$ and $\frac{11}{3}$.
8. Convert $2.\overline{743}$ in the form of $\frac{p}{q}$. [Ans: $\frac{1553}{445}$]
9. Convert $3.\overline{337}$ in the form of $\frac{p}{q}$. [Ans: $\frac{751}{213}$]
10. locate $\sqrt{10}$ on the number line.

Ch-2 Polynomials

1. For what value of k , $x=4$ is a zero of the polynomial $2x^3 - 3x^2 - 18x + k$? [Ans. $k = -8$]
2. Check whether 1 and 2 are zeroes of the polynomial $2x^4 - 6x^3 + 3x^2 + 3x - 2$. [Ans. Yes]
3. Using remainder theorem, find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by $x - \frac{1}{2}$. [Ans. $\frac{27}{8}$]
4. If $x-4$ is a factor of $5x^3 - 7x^2 - kx - 28$, find the value of k . [Ans. $k = 45$]
5. Factorise $6x^2 + 5x - 6$. [Ans. $(2x+3)(3x-2)$]
6. Factorise $9x^2 - 3x - 2$. [Ans. $(3x-2)(3x+1)$]
7. Using factor theorem, factorize $x^3 - 2x^2 - x + 2$. [Ans. $(x-1)(x-2)(x+1)$]
8. Using factor theorem, factorize $4x^3 + 20x^2 + 33x + 18$. [Ans. $(2x+3)(x+2)(2x+3)$]
9. Expand $(3a + 4b + 5c)^2$.
10. Factorize $8a^3 - b^3 - 12a^2b + 6ab^2$. [Ans. $(2a-b)(2a-b)(2a-b)$]